Dumbbell dipole model and its application in UXO discrimination

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ABSTRACT

Electromagnetic Induction (EMI) is one of the most promising techniques for UXO discrimination. Target discrimination is usually formulated as an inverse problem typically requiring fast forward models for efficiency. The most successful and widely applied EMI forward model is the simple dipole model, which works well for simple objects when the observation points are not close to the target. For complicated cases, a single dipole is not sufficient and a number of dipoles (displaced dipoles) has been suggested. However, once more than one dipole is needed, it is difficult to infer a unique set of model parameters from measurement data, which is usually limited.

Inspired by the displaced dipole model, we developed the dumbbell dipole model, which consists of a special combination of dipoles. We placed a center dipole and two anti-symmetric side dipoles on the target axis. The center dipole functions like the traditional single dipole model and the two side dipoles provide the non-symmetric response of the target. When the distance between dipoles is small, this model is essentially a dipole plus a quadrupole. The advantage of the dumbbell model is that the model parameters can be inferred more easily from measurement data. The center dipole represents the main response of the target, the side dipoles act as additional backup in case a simple dipole is not sufficient. Regularization terms are applied so that the dumbbell dipole model automatically reduces to the simple dipole model in degenerate cases. Preliminary test shows that the dumbbell model can fit the measurement data better than the simple dipole model, and the inferred model parameters are unique for a given UXO. This suggests that the model parameters can be used as a discriminator for UXO. In this paper the dumbbell dipole model is introduced and its performance is compared with that of both the simple dipole model and the displaced dipole model.

Keywords: Electromagnetic Induction (EMI), dipole model, dumbbell, UXO, inversion, quadrupole, regularization

1 INTRODUCTION

Fast (real time) data processing approaches for UXO discrimination are essential to reduce the cost of UXO field remediation. UXO detection and discrimination is basically an inverse problem in which the target information is inferred from measurement data, by fitting with some forward models. The simple dipole model [**1**,**2**,**3**], in which one approximates a target's response with an infinitesimal magnetic dipole, is by far the most successful and widely employed forward EMI Electromagnetic Induction (EMI) model, mostly because of its simplicity and associated low computational cost.

However, researchers have begun to realize the limitations of simple dipole model. The dipole approximation is only valid when the observation point is far enough away from the target. The distance required depends on the complexity of the target. In general, most UXOs are highly heterogeneous and complicated in geometry, and the observation points are sometimes very close to the target. In these difficult cases, a simple dipole is not sufficient to describe the complicated response of the target. Nevertheless, we can still apply the simple dipole and find the optimal equivalent polarizability matrix by fitting the measurement data. The problem is that the fitting will be poor and the inferred

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equivalent polarizability matrix will not be unique for a given target. The inferred value of model parameters will vary depending on which part of data we want to fit better.

For these complicated cases, a single dipole is not sufficient and a more flexible model based on a number of dipoles (displaced dipoles) has been suggested. However, the real field data is usually very limited and a simple dipole is sufficient in most cases. The displaced dipole model is only needed in a small number of cases or at some specific frequencies. Applying the displaced dipole model in simple situations will cause over fitting and result in non-uniqueness difficulty. To overcome this disadvantage, we developed a modified model called the dumbbell dipole model, which has the flexibility of the displaced dipole model but automatically reduces to the simple dipole in degenerate cases.

In Section 2 of this paper we will introduce the formulation of the simple dipole model, the displaced dipole model and the dumbbell dipole model. Section 3 will consist of examples showing the inadequacy of the simple dipole model and the better performance of the dumbbell dipole model. The difficulties associated with the displaced dipole model in degenerate cases are also discussed. Section 4 includes our conclusions and discussion of the possibility of using oversimplified model for UXO discrimination.

2 FORWARD MODELS AND FORMULATIONS

2.1 Simple dipole

In the simple dipole model, we approximate a target's response with a single magnetic polarizability matrix, which behaves like a dipole when excited by a primary field. The scattered field can be calculated as

$$\mathbf{H}^{s} = \frac{1}{4\pi\left|\mathbf{R}-\mathbf{R}_0\right|^3}\left(\frac{3(\mathbf{R}-\mathbf{R}_0)(\mathbf{R}-\mathbf{R}_0)}{\left|\mathbf{R}-\mathbf{R}_0\right|^2}-\overline{\overline{\mathbf{I}}}\right)\cdot\mathbf{M}\cdot\mathbf{H}^{\mathbf{pr}}$$

where $\mathbf{R}_0=(x_0,y_0,z_0)^T$ is the position of the dipole or the object and $\mathbf{R}=(x,y,z)^T$ is the observation point where the scattered field is measured. In the principle target coordinate system, matrix M is diagonal: $\mathbf{M}=diag\left[\beta_x,\beta_y,\beta_z\right]$

Most UXOs are Body of Revolution (BOR), in which case the model parameters are $x_0,y_0,z_0,\theta,\phi,\beta_r,\beta_z$. Its axis direction is defined by the Euler angles θ,ϕ and its polarizability matrix becomes $\mathbf{M}=diag\left[\beta_r,\beta_r,\beta_z\right]$.

The biggest advantage of dipole model is that it is simple and requires very low computation cost. The simple dipole model is by far the most widely employed model.

2.2 Displaced dipole model

An inherent assumption in the simple dipole model is that the object is symmetric from both ends, which is usually not true for general targets like UXOs. To account for the asymmetry property, researchers at Duke University [**2**] suggested the displaced dipole model, i.e. a number of independent dipoles placed on the axis of a BOR object.

In the case of two dipoles, we place the two dipoles on the symmetric axis of the BOR object. We define the middle of the two dipoles as the model center (x_0,y_0,z_0). The orientation of the object (and the dipoles) is defined by angles θ,ϕ. The distance between the two dipoles is defined by a positive parameter p. So the model parameters for the displaced dipole model with two dipoles are $x_0, y_0, z_0, \theta, \phi, p, \beta_{r0}, \beta_{z0}, \beta_{r1}, \beta_{z1}$, where $\beta_{r0}, \beta_{z0}, \beta_{r1}, \beta_{z1}$ are the magnitude of the two dipoles. The target response is simply the summation of the two dipole response.

2.3 Dumbbell dipole model

Another idea to account for UXO complexity is to use higher poles (i.e. quadrupoles, etc.). Inspired by these two different mentalities, i.e. the displaced dipole model and the use of higher order poles, we developed a model that we call the dumbbell dipole model. The dumbbell dipole model is specifically designed for BOR such as most UXOs. It consists of a special combination of dipoles (i.e. equivalent polarizability matrices): a center dipole M_0 and two anti-symmetric side dipoles, all on the axis of the BOR object (Figure 1). The center dipole M_0 functions like the traditional single dipole. The two side dipoles have the same distance p from the center dipole, with polarizability matrices M_1 and $-M_1$. The side dipoles provide the non-symmetric response of the target. When the distance p is small, this model is essentially a dipole plus a quadrupole. The advantage of the dumbbell model over the displaced dipole model is that the model parameters can be inferred more easily from measurement data. The center dipole represents the main response of the target, the side dipoles act as additional backup in case a simple dipole is not sufficient. A penalty term is added to ensure that the dumbbell dipole model automatically reduces to simple dipole model in degenerate cases.

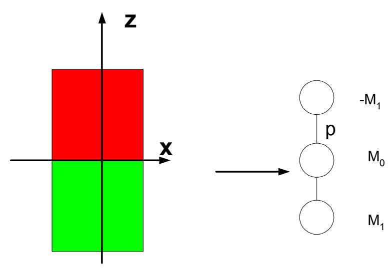

Figure 1 Dumbbell dipole model

According to the dumbbell dipole model, the scattered field of an object can be represented as

$$\mathbf{H}^s = \frac{1}{4\pi|\mathbf{R}-\mathbf{R}_0|^3}\left(\frac{3(\mathbf{R}-\mathbf{R}_0)(\mathbf{R}-\mathbf{R}_0)}{|\mathbf{R}-\mathbf{R}_0|^2} - \bar{\bar{\mathbf{I}}}\right)\cdot \mathbf{M}_0 \cdot \mathbf{H}^{pr}(\mathbf{R}_0) + \frac{1}{4\pi|\mathbf{R}-\mathbf{R}_1|^3}\left(\frac{3(\mathbf{R}-\mathbf{R}_1)(\mathbf{R}-\mathbf{R}_1)}{|\mathbf{R}-\mathbf{R}_1|^2} - \bar{\bar{\mathbf{I}}}\right)\cdot \mathbf{M}_1 \cdot \mathbf{H}^{pr}(\mathbf{R}_1) - \frac{1}{4\pi|\mathbf{R}-\mathbf{R}_2|^3}\left(\frac{3(\mathbf{R}-\mathbf{R}_2)(\mathbf{R}-\mathbf{R}_2)}{|\mathbf{R}-\mathbf{R}_2|^2} - \bar{\bar{\mathbf{I}}}\right)\cdot \mathbf{M}_1 \cdot \mathbf{H}^{pr}(\mathbf{R}_2)$$

where all dipoles are on the axis of the object.

To avoid over fitting and non-uniqueness problem in degenerate cases, we apply regularization to ensure stable model parameters. The regularization is designed so that the model will converge into a simple dipole model in degenerate cases.

The objective function is $F(\rho) = \dfrac{\sum_{\mathrm{j}} w_{\mathrm{j}}^2 \left(H_{\mathrm{j}} - H_{\mathrm{j}}^{\mathrm{d}}\right)^2}{\sum_{\mathrm{j}} w_{\mathrm{j}}^2 \left(H_{\mathrm{j}}^{\mathrm{d}}\right)^2} + \alpha p + \beta \dfrac{|M_1|}{|M_0|}$

where α and β are empirical regularization parameters to ensure reasonable bounds on p and M_1. In the model fitting process, we start with strong constraint (big α and β values) and gradually release them till we obtain the desired goodness of fit. w_j is the weighting function and usually chosen to be 1.

3 EXAMPLES AND RESULTS

In this section we will study some examples showing the insufficiency of the simple dipole model and better performance of the dumbbell dipole model.

3.1 Simple Composite cylinder

First we consider a composite object where one magnetic steel cylinder is connected with an aluminum cylinder (14cm long in total and 3.8cm in diameter). GEM measurements were done at 8 points along the axis with positions z1=33cm, z2=27cm, z3=22cm, z4=17cm, z5=-17cm, z6=-22cm, z7=-27cm, z8=-33cm, as shown in Figure 2.

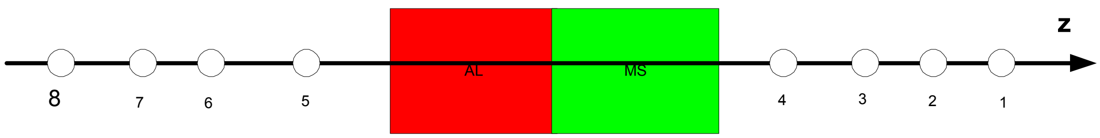

Figure 2 Object and observation points

Below we fit the data with different kinds of forward models.

3.1.1 Simple dipole model #1

In most applications of dipole models, one usually assumes that the dipole is located at the center of the target [4,5]. Here we start with this simple form of dipole model, fixing the dipole position at the center of object ($z_0 = 0$),

This is an axial case, so only one component (β_z) is considered, which can be calculated directly from

$$\mathbf{H}^s = \frac{1}{4\pi\left|z-z_0\right|^3}\left(3\hat{z}\hat{z}-\overline{\overline{\mathbf{I}}}\right)\cdot\beta_z H_z^{pr}\hat{z} = \frac{1}{2\pi\left|z-z_0\right|^3}\beta_z H_z^{pr}\hat{z} \quad \text{i.e.} \quad \beta_z = \frac{H^s\ 2\pi\left|z-z_0\right|^3}{H_z^{pr}}$$

The real and imaginary parts of β_z at each frequency can be calculated directly from measurement data H^s.

The calculated β_z values are plotted in Figure 3. The results show that using only the data at each different observation points results in different β_z. There is no single unique β_z associated to the given target. The required β_z depends on both frequency and observation position.

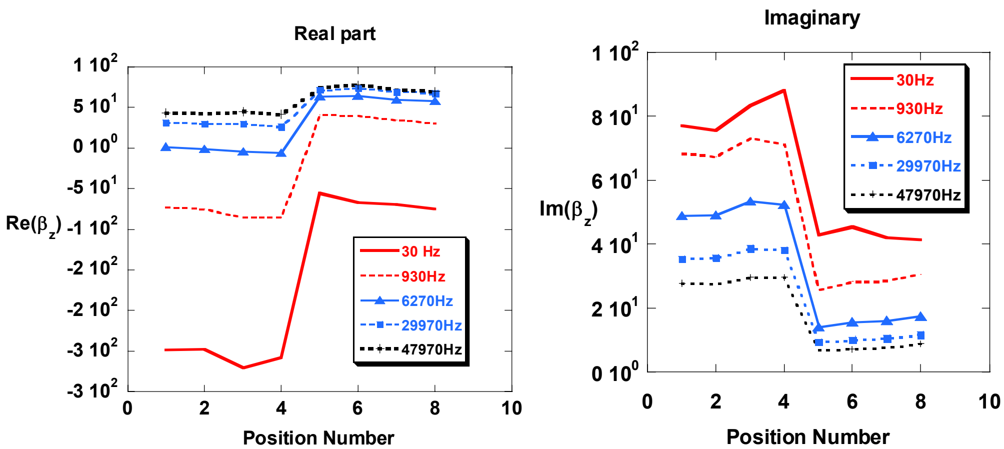

Figure 3 inferred dipole moments from data at each of the 8 measurement positions

3.1.2 Simple dipole model #2

Fixing the dipole position at the target center might be too much of a constraint. Accordingly, in this model we allow the dipole position (z_0) to be another model parameter and fit all 8 data observations with one simple dipole. The free model parameters are z_0 and β_z . The model fitting were done at each frequency component by minimizing the cost function $f_{obj}(f)=\sum_{j=1}^{8}\left(H_j^d(f)-H_j^m(f)\right)^2$ for each frequency f, real or imaginary component. The inferred model parameters are plotted in Figure 4.

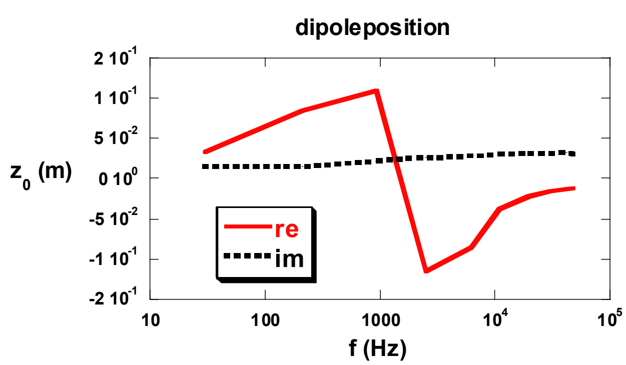

Figure 4 dipole position derived from measurements at positions 1 to 8

We define the matching factor as $f_{match}(f)=1-\frac{\sum_j\left(H_j^d(f)-H_j^m(f)\right)^2}{\sum_j\left(H_j^d(f)\right)^2}$, which is plotted in Figure 5.

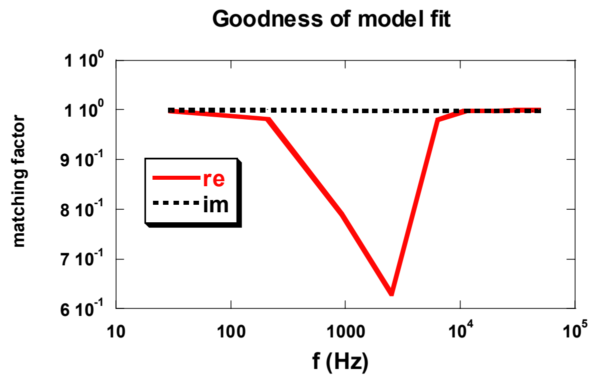

Figure 5 goodness of fit (matching factor)

We see a big jump of model parameters and poor matching at 930Hz and 2490Hz. Comparison of modeled data with measurements reveals the reason.

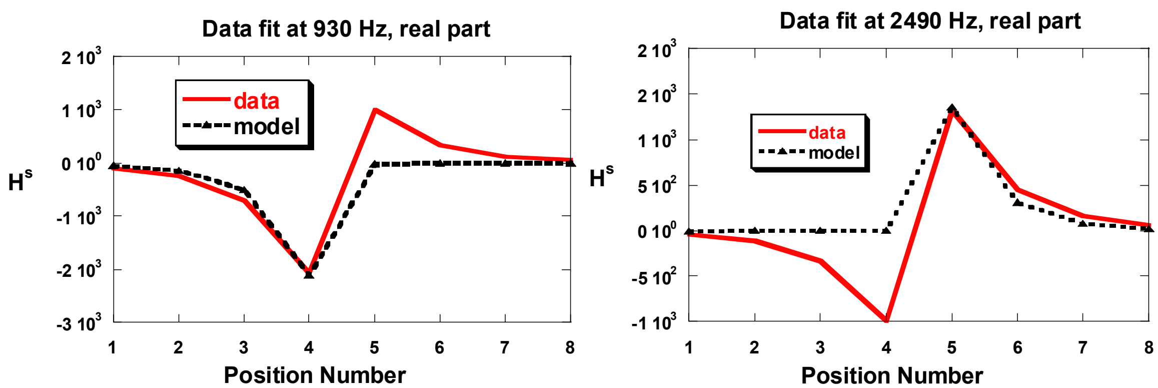

Figure 6 optimal model fit at 930Hz and 2490Hz

The measurement data at 930Hz and 2490Hz change sign as the observation position moves from one end of the target to the other end. A simple dipole cannot support sign changing, so it has to pick the data with one sign (positive or negative only) to satisfy and sacrifice the data with opposite sign. At 930Hz the negative-sign part of the data dominates; in order to reduce the effect of data with positive sign, the model dipole has to be far away from those measurement positions (giving almost zero response at these points, which is the best the model can do), so the position is a big positive number (close to the end of the MS cylinder). At 2490Hz the positive part of the data dominate, so the inferred model produce positive data and the dipole position is a big negative number (close to the end of AL cylinder).

The non-uniqueness issue makes the target discrimination process much more complicated. The inferred dipole model parameters are still informative since in general the values only vary in a certain range, but it adds in some extra uncertainties, which makes it impossible to make a clear conclusion in some discrimination cases. Our goal is to reduce this uncertainty.

3.1.3 Dumbbell dipole model

In Figure 7 and Figure 8, we fit 8 points data with dumbbell dipole at target center position and optimal p. Results show that the dumbbell dipole model can describe target response better than a simple dipole model.

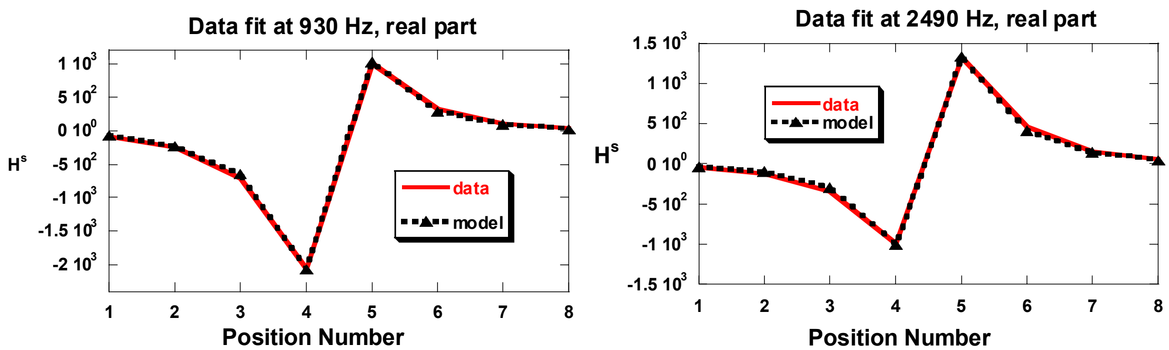

Figure 7 model fit at 930Hz and 2490Hz

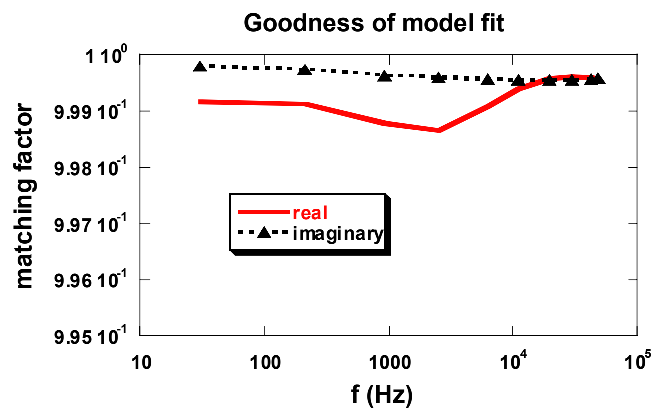

Figure 8 goodness of fit (matching factor)

Figure 9 UXO Heat Round, showing complicated geometry and material composition

3.2 Study of UXO models

We consider a general case which we typically encounter in real field UXO discrimination. UXO Heat Round (one of the most complicated UXO we have in the library, see **Figure 9**) was buried under the ground with center at depth 10.5cm. The measurements were done at a "+" pattern grid, with 7 points on each line. Three grids were measured, with elevation 10cm, 17.5cm and 25cm from the ground surface. Two scenarios were studied: UXO buried horizontally and 45 degree tilted nose down. The data were fit with a simple dipole model (assuming BOR geometry), in which the model parameters include non-frequency-dependent dipole location and orientation $\left(x_0, y_0, z_0, \theta, \phi\right)$, and a frequency-dependent diagonal polarizability matrix $\mathbf{M} = diag\left[\beta_{r0},\ \beta_{r0},\ \beta_{z0}\right]$. Figure 10 shows the goodness of the model fitting, which is more than 85% at all frequency components. At most frequencies the matching is over 95%.

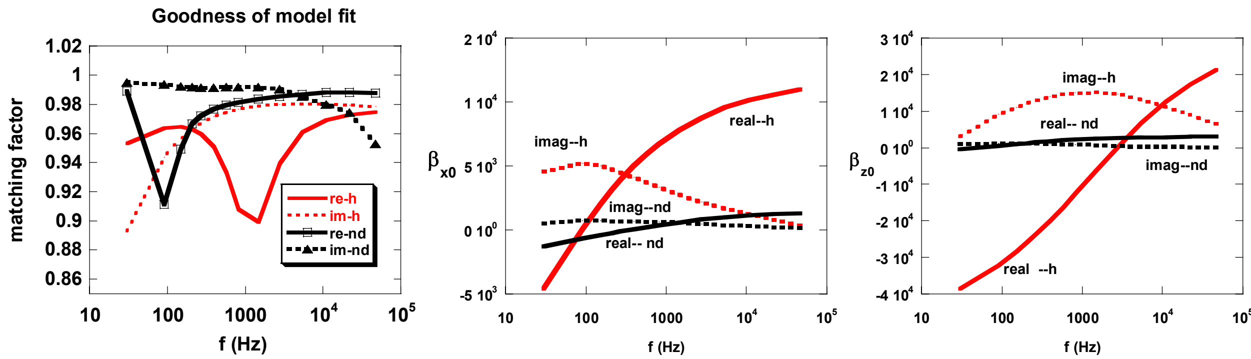

Figure 10 Simple dipole model fitting measurements of UXO. The model fitting is good for the two cases but the inferred model parameters are totally different: "h"= horizontal and "nd"=45^0 tilted nose down

The simple dipole model fits the data fairly well except at several specific frequency components. The results seem to contradict to what we showed in Section 2 but they actually do not. Sections 2 and 3 consider two different kinds of problems. In Section 2, one tries to describe the whole object with a simple dipole, which turned out to be impossible. In this section one tries to deal with real field data, which is always limited. The UXO buried under the ground has a fixed location and orientation, and we can only perform measurements above the ground. To fit this limited data, a dipole model is already sufficient in many cases. This is the reason why simple dipole model is still widely and successfully employed in real field applications.

However, we do see the limitation of the simple dipole model. There is no single unique set of model parameters for a given target. The dipole parameters (β_{x0} and β_{z0}) vary from case to case. The β_{x0} and β_{z0} values for the horizontal case are completely different from those obtained for the tilted case, as shown in Figure 10. The non-uniqueness of model parameters introduces extra difficulties for UXO discrimination and the main cause is the limitation of field measurement data. Along with the development of new instruments and better measurement design, we expect more complete measurement data where the advantage of the dumbbell dipole model over the simple dipole model will be better recognized.

Even with currently available data, the results (i.e. in Figure 10) already show some mismatch in certain frequency components. This mismatch is an indication of insufficiency of the simple dipole model. To compare the performance of the three different dipole models, i.e. the simple dipole model, the displaced dipole model and the dumbbell dipole model, we focus on the horizontal case and pick two frequency components (real part of 1470Hz and real part of 10950Hz) as examples. The simple dipole model works well when considering the real part of 10950Hz data but has difficulties when considering the real part of 1470Hz data. The model fitting was done for each individual frequency component. To study the stability of model fitting, we pick groups of data from the whole set of 42 measurement points. The 42 data points are first sorted according to their total magnitude, i.e. $\sum_f \left|H^d(f)\right|$. Group number 1 is the whole set of data (42 points). To construct group number 2, we simply take out the 3 points with the biggest signal magnitude $\sum_f \left|H^d(f)\right|$, so we have 39 points in group 2. Similarly, group number n has $42-3(n-1)$ points, with the $3(n-1)$ points with strongest signal missing. As the group number increases, the number of data points left decreases and the signal gets weaker. The inferred dipole parameters and matching factors for different groups are plotted in Figure 11 to Figure 14. We studied the simple dipole model, the displaced dipole model (with 2 dipoles) and the dumbbell dipole model.

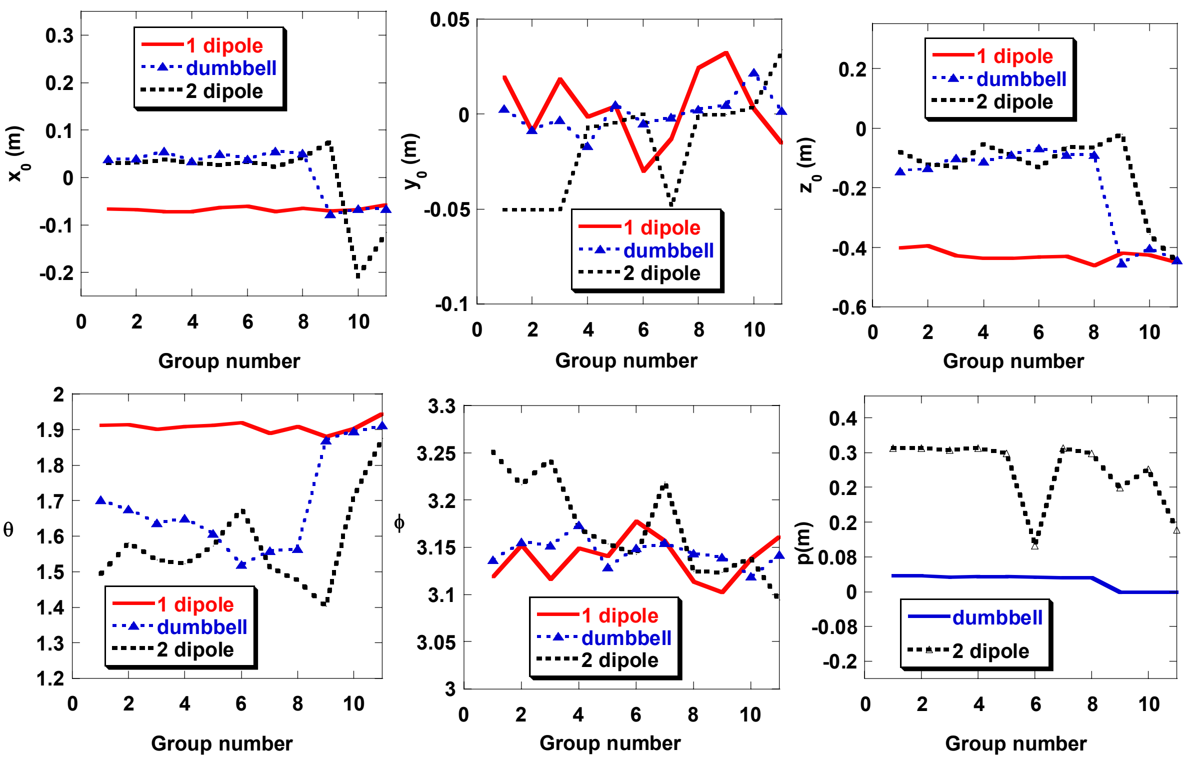

Figure 11 Inferred dipole positions for 1470Hz

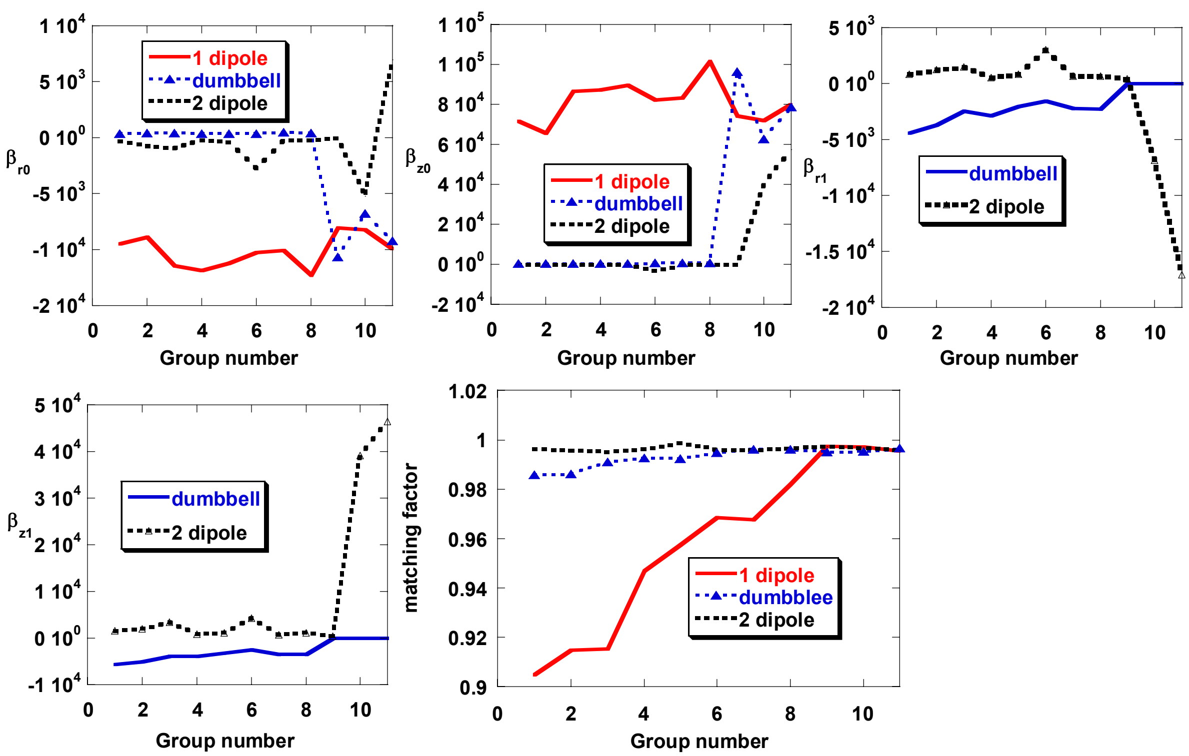

Figure 12 Inferred β_{x0}, β_{z0}, β_{x1}, β_{z1} and matching factors for real part of 1470Hz

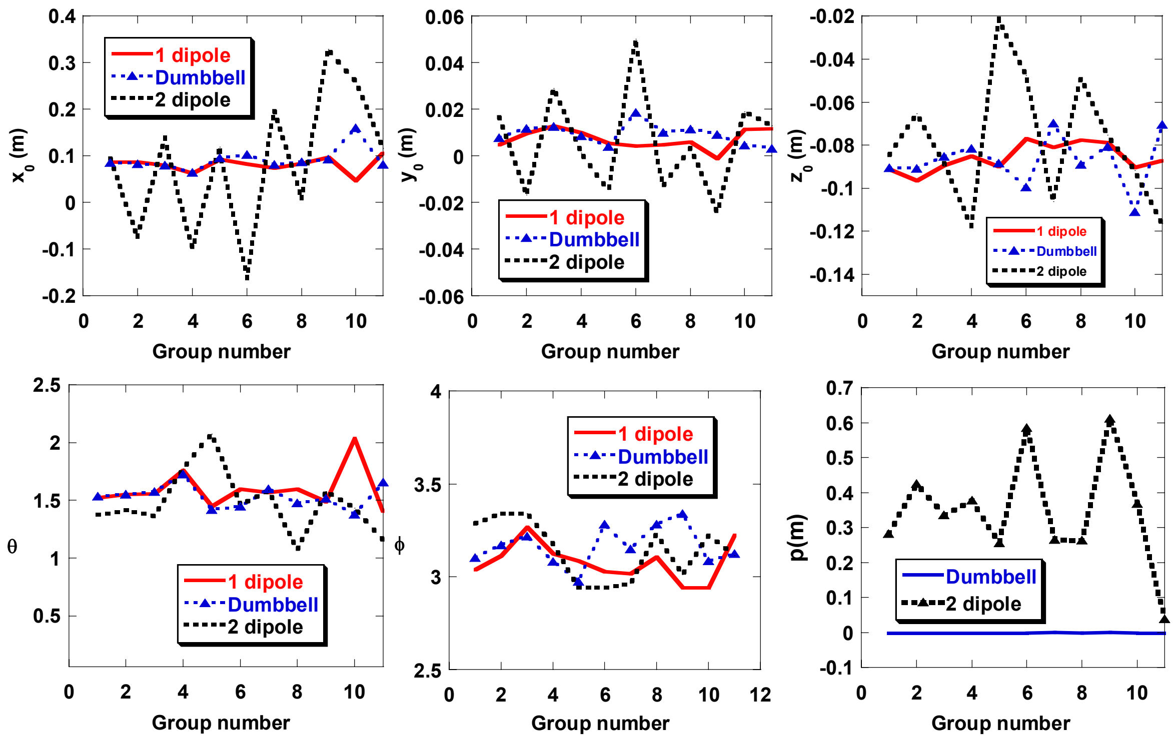

Figure 13 Inferred dipole positions for 10950Hz

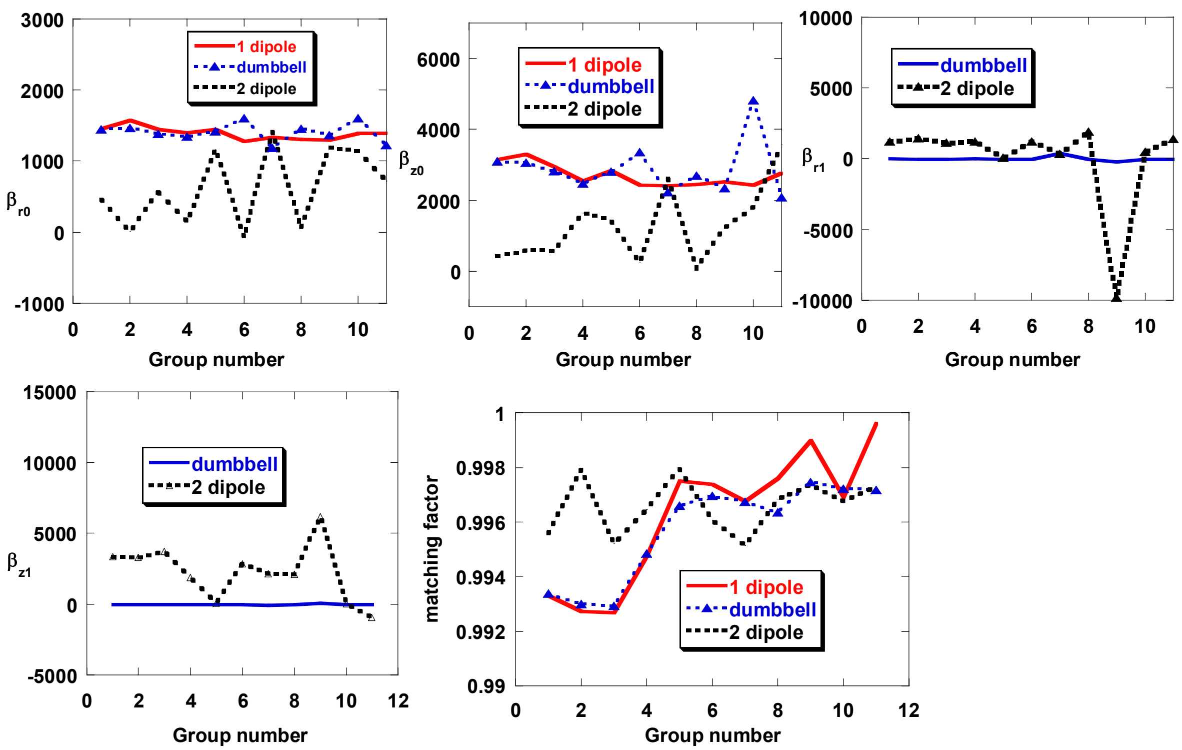

Figure 14 Inferred β_{x0}, β_{z0}, β_{x1}, β_{z1} and matching factors for 10950Hz

A simple dipole is not sufficient to describe the data at 1470 Hz (matching factor ~90%). Both displaced dipole model and the dumbbell dipole give much better fits. For different groups of data, inferred model parameters for the simple dipole model and the dumbbell dipole are stable although their values are different. At high group numbers, where the data is limited, a single dipole becomes sufficient. In the high group number cases, the dumbbell dipole model reduces to a single dipole model, which causes a big jump from the dumbbell solution to the simple dipole solution. The displaced dipole model is stable in most cases but the position of the second dipole varies considerably.

At 10950Hz the simple dipole model is sufficient and all three models produce good fits; the dumbbell solution converges to the simple dipole. But the displaced dipole becomes unstable. Inferred model parameters are not unique for different groups of data, which make it difficult to use these parameters as discriminators.

4 CONCLUDING DISCUSSION

In this paper we studied the performance of three kinds of dipole models: the simple dipole model, the displaced dipole and the newly developed dumbbell dipole model. The simple dipole model is widely used because of its low computation cost. However, it is not sufficient to describe the target response in some complicated cases. The displaced dipole model is more flexible but it will encounter non-uniqueness difficulty in degenerate cases, i.e. in simple cases where a single dipole model is already sufficient to fit the data. The dumbbell dipole model also has the capability of fitting more complicated data. Its specially designed regularization term also ensures that it will automatically reduce to the simple dipole model in degenerate cases, without causing ill-conditioning problems.

The complete EMI response of a general UXO is complicated and dipole models may not be able to fully describe it. However, in real field conditions, the UXO is buried under the ground at a fixed position and orientation. The data we can measure is very limited. For state of the art EMI instruments with current measurement designs, a simple dipole model can fit the measurement data in most cases and a dumbbell dipole model is essentially sufficient for real practical measurement data. The limitations of measurement data are currently the major bottleneck in UXO discrimination.

With the development of new instruments or better measurement designs, the limitation of dipole models may become more obvious, in which case more sophisticated forward models maybe needed. However, simple models are still attractive especially because of their low computational cost. In complicated cases, dipole models become oversimplified. The inferred model parameters will vary depending on which part of the data we pick for inversion or what weighting functions we apply.

The non-uniqueness of inferred model parameters makes discrimination more complicated. Instead of getting a unique set of model parameters, we now get “a cloud” of sets. These inferred model parameters, although not unique, are still informative and can be used for discrimination. The model parameters usually vary within a certain range and this range can be used to estimate target information. The parameters vary according to frequency and groups of data used for model fitting. Wild changing of parameters is usually a positive sign of a complicated object (like UXO). The changing pattern can also be used for discrimination. As an example, Figure 4 showed the variation of inferred dipole position changing along frequencies. The model parameters had a big jump big jump with poor fitting at 930Hz and 2490Hz, from which we can conclude that this is a complicated object. The inferred dipole center close to one end at low frequency and moves to the other end at higher frequency, which indicates that the first end is highly magnetic and the other end is less or non magnetic. In general, the pattern information may not be so clear like this, but a statistical approach or machine learning approach might be able to extract valuable information from the changing pattern.

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